MATH 121A Prep: Proofs

1. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Prove that for all positive integers n, $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

Solution: Since we want to show something is true for all positive integers we will do a proof by induction.

Base Case - $n = 1$: $A^1 = A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ which is of the correct form.

Inductive Step: Suppose that this is true for a particular n. Then

$$A^{n+1} = AA^n = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & n+1 \\ 0 & 1 \end{bmatrix}$$

as desired.

Therefore the statement is true for all positive n.

2. Convert the following statements between words and mathematical notation.

   (a) For all $\vec{v}$ in V there exists unique $\vec{u}$ in U and $\vec{w}$ in W such that $\vec{v} = \vec{u} + \vec{w}$.

   (b) $\exists \vec{v} \in V$ such that $\forall \vec{w} \in W$, $f(\vec{v}, \vec{w}) = 0$.

Solution: (a) $\forall \vec{v} \in V \; \exists! \; \vec{u} \in U$, $\vec{w} \in W$ such that $\vec{v} = \vec{u} + \vec{w}$.

(b) There exists $\vec{v}$ in V such that for all $\vec{w}$ in W, $f(\vec{v}, \vec{w}) = 0$.

3. Write the negation of the statements in Question 2.

Solution: (a) There exists $\vec{v}$ in V such that for all $\vec{u}$ in U and $\vec{w}$ in W, $\vec{v} \neq \vec{u} + \vec{w}$.

(b) $\forall \vec{v} \in V \; \exists \vec{w} \in W$ such that $f(\vec{v}, \vec{w}) \neq 0$.